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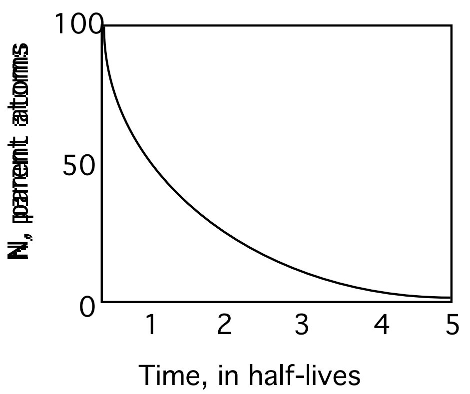
Radioactivity

Radiometric Dating

The principle of radiometric, or absolute age, dating is critical to much of what we have learned about the history of the solar system, planet Earth and our own civilization. It is therefore important to understand how radiometric dating works. Recall that atoms of a particular element have a characteristic number of protons. But the nuclei of atoms of one type may contain different numbers of neutrons, and atoms that differ in this way are called *isotopes* of an element. Due to varying configurations of protons and neutrons, some nuclei are unstable and break down through the process of radioactive decay, forming daughter atoms of a different element. During radioactive decay, various forms of radiation are emitted. What is particularly important about radioactive decay is that it occurs at a known rate for any particular atom, and thus the number of atoms at a given time is related to the original number of atoms as well as this known rate. We generally quantify this rate with the term “half-life.” The "half-life" of a radioactive isotope is the amount of time it takes for half of a population of "parent" atoms to break down into a daughter product. To help visualize this, here is a plot of the proportion of original parent atoms remaining after each successive half-life:



The subject of radioactivity has natural links with math topics, at lower grade levels, graphing, and at higher grade levels, exponential equations, algebraic manipulations, and integration (though we won’t get into calculus here).

This curve describes the decay of a radioactive isotope and it is expressed mathematically by the following relationship:

$$N = N_0 e^{-\lambda t} \qquad (1)$$

where: No = the original number of atoms of the radioactive element
λ = the decay constant, a specific number for each element
t = time in years
N = the number of unchanged atoms remaining at time t

Equation (1) is the basic equation that presents radioactive decay.

Let's further define this decay constant (λ) in terms of the half life of the radioactive element. At t = one half life = $t_{1/2}$, half of the original atoms have decayed to daughter products. Thus, N = 1/2 N_o. Substituting this into equation (4-1), we have:

$$\frac{1}{2}N_o = N_o(e^{-\lambda t_{1/2}}) \qquad \textbf{(2)}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$\ln 2 = \lambda t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Now, assume at time zero (when a mineral forms, say it cools from a melt and traps atoms of a radioactive parent atom) that no daughter products are present. The number of daughter atoms at any time (D_t) is given by:

$$D_t = N_o - N \qquad (3)$$

We can rearrange the basic decay equation given above (1) to obtain:

$$N_o = Ne^{\lambda t} \qquad (4)$$

Substituting into the equation for daughter atoms (3), this becomes:

$$D_t = Ne^{\lambda t} - N = N(e^{\lambda t} - 1) \qquad (5)$$

In the more general form, the number of daughter atoms at any time is given by:

$$D = D_o + D_t \qquad (6)$$

where D_o = some initial amount of the daughter isotope contained in the rock at time zero.

We can rearrange this by combining (5) and (6) to get:

$$\mathbf{D = D_o + N(e^{\lambda t} - 1)} \qquad \textbf{(7)}$$

This is the basic equation for dating rocks and minerals. Provided that a good estimate of the original amount of daughter atoms (D_o) can be made (this is done by measuring ratios of various isotopes and daughter products), analysts can measure D and N in the laboratory and solve for t that equals the age of the system of interest.

There are several important conditions that must be met in order for this process to be valid. They are:

1. The system must have remained closed with respect to daughter products (e.g. no loss or contamination).
2. A realistic value for D_0 must be obtained.
3. The decay constant, λ (lambda), must be known. It is determine experimentally.
4. Values of D and N, measured in the lab, must be accurate.

Different radioactive elements have different half lives and thus depending on the time scale of interest, different radioactive systems are used for age determinations. For instance, ^{14}C has a half-life of 5730 years and is generally good for dating carbon-bearing materials back to about 40,000 years old. An isotope of uranium (^{238}U), on the other hand, has a half-life on the order of 10^{10} years; it is thus good for dating the oldest samples available like meteorites or the earliest terrestrial rocks, lunar rocks, etc.

EXERCISES

(1) Place all candies in a plastic cup. Shake and then pour onto the surface of the table in a single layer. Take care not to pour the candies so vigorously that they go spinning under the table. Recall that mass must be conserved.
(2) Separate the candies into "label-side up" and "label-side down.
(3) Count and record the number of candies in each group and record your observation in the attached datasheet.
(4) <u>Remove</u> the label-side up candies from the system
(5) Repeat the cycle of steps 1-4 until you have 0 or 1 candy left.

Consider each step that you removed candies as one "half-life" of the system.

A. How many half-lives did you observe?
B. Add up the total number of candies "removed" from the system. How many candies did you have when you started? Make a plot of your observations. For each "half-life," plot the number of candies label-side-down in the system.

Note that in real systems, whether or not a radioactive parent decays is something that is determined by probability. The numbers of atoms are much larger, however, and it generally works out that nearly (but not always exactly) half the atoms present of a radioactive element decay over a half-life.).

Follow-up questions

1. Assume your "half-lives" you manufactured in class represented decays of a nuclide since the formation of the solar system to the present day. How long would each of your half-lives represent in earth-years?

2. Calculate the age of a rock (i.e. a chemical system) given the following information. A mineral growing in a rock as the rock cooled from a melt contains a radioactive nuclide N whose half-life is 250 million years; the measured ratio of D/N at present is 1.75. Assume that $D_o = 0$.

In-Class Exercise "Worksheet"

(50 points - turn this in with your lab write-up)

Step (x-axis)	# label-side up	# label side down (y-axis) (these are kept in the system)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10